Anderson Theorem for Color Superconductor

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Abstract

Ginzburg-Landau functional is derived for a system possessing both chiral and diquark condensates. Anderson theorem for such a system is formulated and proved.

1 Introduction

During the last couple of years color superconductivity became a compelling topic in QCD - see e.g. the review papers [1, 2]. The present work is aimed to shed some new light on the region of mixed condensation in which both chiral and color symmetries are broken. For definiteness we consider the case of two flavors u and d, the corresponding color superconducting phase is called 2SC [1, 2]. The point at issue is the interplay of the chiral condensate φ and the diquark condensate Δ in the interval of the chemical potential μ where they possibly coexist. Necessary to mention that as model calculations show [3]-[6] the very presence of such a region is questionable. Instead of coexistence one rather observes a strong competition between φ and Δ in the sense that where one condensate is nonzero the other vanishes. This conclusion has been drawn as a result of numerical analysis of the complicated set of two coupled gap equations for φ and Δ [3, 5] (other authors take it as a plausible apriori assumption [4]). The question has been also addressed within the random matrix approach [7] with the result that for single-gluon exchange the coexistence region of the two condensates is absent. Evidently such a situation calls for simplification. Arguments presented below are aimed to show that as soon as (with μ increasing) the diquark condensate Δ develops the role of the chiral condensate φ is prescribed by what may be called the Anderson theorem [8, 9]. In BCS theory this theorem states that nonmagnetic impurity do not influence the thermodynamic properties of a superconductor (in particular the values of the gap and of the critical temperature) in linear approximation in impurity concentration. In case of color superconductor the role of impurity is taken by chiral condensate. This means that the system of the gap equations for φ and Δ actually decouples and with high accuracy one can calculate Δ neglecting the influence of φ. One general remark is probably needed. Namely, the content of the "true" Anderson theorem in BCS is physically deeper than the statements formulated below. This point has to be discussed within comparative study of BCS theory and color superconductivity which is beyond our present goals (see [1,2]).

In Section 2 we present and discuss the general expression for thermodynamic potential which has the structure common to three approaches to color superconductivity, namely NJL, one gluon exchange, and instanton ensemble. In Section 3 we derive the Ginzburg-Landau functional for chiral condensate, while in Section 4 the same is done for that part of the potential which depends on both condensates - the diquark and the chiral ones. The main result of this Section is that the contribution of the chiral condensate to the "mixed" part of thermodynamic potential may appear only in orders higher than second one. In Section 5 we show that the same is also true at zero

temperature. Section 5 is devoted to the formulation and discussion of what we call the Anderson theorem for color superconductor.

2 Partition Function and Thermodynamic Potential

We start with the QCD Euclidean partition function

$$Z = \int DAD\bar{\psi}D\psi \exp(-S), \tag{1}$$

where

$$S = \frac{1}{4}\int F^a_{\mu\nu}F^a_{\mu\nu}d^4x = \int \bar{\psi}(-i\gamma_\mu D_\mu - im + i\mu\gamma_4)\psi d^4x. \tag{2}$$

In (2) color and flavour indices are suppressed, $N_f = 2,\ N_c = 3$, and the chemical potential μ is introduced. Performing integration over the gauge fields one gets effective fermion action in terms of cluster expansion

$$Z = \int D\bar{\psi}D\psi \exp(-\int d^4xL_0 - S_{eff}), \tag{3}$$

with $L_0 = \bar{\psi}(-i\gamma_\mu\partial_\mu - im + i\mu\gamma_4)\psi$ and effective action $S_{eff} = \sum_{n=2}^{\infty}\frac{1}{n!}\langle\langle\theta^n\rangle\rangle$, where $\theta = \int d^4x\bar{\psi}(x)g\gamma_\mu A^a_\mu(x)t^a\psi(x)$ and double brackets denote irreducible cumulants [10]. Since we consider only u and d quarks the current quark mass m will be neglected. In future we plan to give up this approximation especially in view of the numerical results obtained in [3] which show that there is some influence of m on the phase structure near the transition point. One should also keep in mind that integration over gauge fields when passing from (1) to (3) is in no way a trivial operation. It certainly deserves dedicated investigation which is outside the scope of the present work (for some guidlines see [11]).

To proceed further some simplification of (3) is needed. Being quite general, (3) gives rise to several commonly used models. First step is to keep in S_{eff} only the lowest four-quark interaction. Routine manipulations then lead to the one gluon exchange model [12] which in turn displays the bulk properties of the color superconductivity phenomena [1]. One gluon exchange yields surprisingly interesting results at very high density [1, 2]. Less trivial but very transparent approach [13] enables to recast (3) into the delute instanton gas model which is successfully used in color surerconductivity studies [14, 5, 1]. We note that derivation of the instanton model presented in [13] goes beyond zero modes approximation. The importance of higher modes was demonstrated in [15, 16]. Simplifying things even further [13] one arrives at the NJL models which as also quite suitable to describe color superconductivity [1, 4].

A lesson from comprehensive studies of color superconductivity within different models listed above is that the main results are very similar [1, 2]. The key technical points leading from chosen Lagrangian to the effective action are similar as well. Use is made of the bosonization of the fields $(\bar{\psi}\psi)$ and $(\psi\psi)$ [3]. As a result the following general expression in the Nambu-Gorkov basis of eight component fields $(\psi, \bar{\psi}^T)$ emerges:

$$S_{eff} = \int d^4x \left\{ \frac{\varphi^2}{4g_1^2} + \frac{\Delta^+\Delta}{4g_2^2} - \frac{1}{2}tr\ln\begin{pmatrix} \Delta\Phi & i\partial_\mu\gamma_\mu + i\varphi\Lambda - i\mu\gamma_4 \\ -i\partial^T_\mu\gamma^T_\mu - i\varphi\Lambda^T + i\mu\gamma_4 & \Delta^+\Phi^+ \end{pmatrix}\right\}. \tag{4}$$

Let us present necessary explanations to (4). We use the following representation

$$\gamma_k = \begin{pmatrix} 0 & -i\sigma_k \\ i\sigma_k & 0 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{5}$$

$$C = \gamma_2\gamma_4, \quad C^{-1}\gamma_\mu C = -\gamma_\mu^T, \partial_\mu^T = \overleftarrow{\partial}_\mu = -\overrightarrow{\partial}_\mu. \tag{6}$$

Effective action (4) is written in terms of the two condensates - the chiral one φ and the superconducting diquark one Δ. Operators Φ and Λ in (4) determine the matrix structure of these condensates

$$\Delta_{ij}^{\alpha\beta} = \varepsilon_{\alpha\beta 3}\varepsilon_{ij}C\gamma_5\Delta \equiv \Phi\Delta, \ \varphi_{ij}^{\alpha\beta} = \delta_{\alpha\beta}\delta_{ij}\varepsilon \equiv \Lambda\varphi, \tag{7}$$

where indices α, β correspond to color, and i, j - to flavor. The matrix structure defined by (7) is the simplest possible. For example, if one starts with the single gluon exchange model and performs Fierz transformation with respect to Lorentz, color and flavor indices, one arrives to several other diquark condensates in addition to (7), e.g. the negative parity one with $\Phi' = \varepsilon_{\alpha\beta 3}\varepsilon_{ij}C$, etc., see [1, 17]. The same is true for the matrix Λ. The minimal "improvement" of Λ in line with chiral invariance requirement would be $\Lambda' = \tau_{a=0-3} + i\gamma_5\tau_{a=0-3}$ while in (7) only the $a = 0$ part of the first term in kept. In other words from the two fields $(\sigma, \boldsymbol{\pi})$ only σ is kept. The two coupling constants g_1^2 and g_2^2 have the dimension of m^{-2}. In the one gluon exchange inspired model they are related to each other but in most studies they are considered as independent [1]. Regularization of the integrals involving the chiral gap is performed either by smooth formfactor or by cutoff. The sensitivity to the regularization procedure is unessential [1] and to avoid complicated equations we do not introduce this regularization explicitly. Integrals involving the superconducting gap Δ are regularized at the Debye frequency –see below.

Factorizing from (4) the four-volume and making use of the identity

$$tr\ln\begin{pmatrix} R & A \\ -A^T & R^+ \end{pmatrix} = tr\ln AA^T + tr\ln\left\{1 + A^{-1}R(A^T)^{-1}R^+\right\} \tag{8}$$

one arrives at the following expression for thermodynamic potential $\Omega(\varphi, \Delta; \mu, T)$

$$\Omega(\varphi, \Delta : \mu, T) = \frac{\varphi^2}{4g_1^2} + \frac{\Delta^+\Delta}{4g_2^2} - tr\ln(-\hat{p} + i\varphi\Lambda - i\mu\gamma_4) - \\ - \frac{1}{2}tr\ln\left\{1 + \frac{\Delta\Delta^+\Phi\Phi^+}{(\hat{p}_+ - i\varphi\Lambda)(\hat{p}_- + i\varphi\Lambda)}\right\}, \tag{9}$$

where $\hat{p} = p_\mu\gamma_\mu, p_\pm = (p_k, p_4 \pm i\mu)$. This form on Ω was first derived in [3].

The remaining part of the paper will be mostly devoted to the investigation of Ω in the Ginzburg-Landau region, i.e. to the expansion of Ω in powers of Δ and φ at temperatures close to the critical one. Obviously the system under consideration possesses two different critical temperatures - the chiral temperature T_φ and the diquark one T_Δ. The situation simplifies if one focuses on the transition region in the (T, μ) plane where the two condensates φ and Δ compete. Numerical calculations show [3] that in the corresponding interval of μ the slope $\frac{dT}{d\mu}$ of the phase transition line is small and a notion of a common critical temperature T_c makes sense. At $T > T_c$ both condensates φ and Δ vanish. Numerically T_c is about 30-40 MeV while the corresponding values of μ lie in the interval of 300-400 MeV [3]. This is the region of the phase diagram which we will be interested in. Consideration will be restricted to the case of homogeneous condensates φ and Δ. Thus the gradient terms will be absent.

We start the derivation with the expansion of the third term in (9) in powers of φ.

3 Chiral Condensate Transition Line

Consider the "φ-part" of the partition function (9), i.e. the functional

$$\Omega_\varphi(\varphi; \mu, T) = \frac{\varphi^2}{4g_1^2} - tr\ln(-\hat{p} + i\varphi\Lambda - i\mu\gamma_4). \tag{10}$$

The equation of Ω_φ proceeds along the standard lines [18]. One has

$$\Omega_\varphi = \frac{\varphi^2}{4g_1^2} + 2N_cN_fT\int d\varphi^2 \sum_n \int \frac{d^3p}{(2\pi)^3}\frac{1}{[(2n+1)\pi T + i\mu]^2 + p^2 + \varphi^2}, \tag{11}$$

where $N_c = 3, N_f = 2$, and the summation is over the fermionic Matsubara modes. After performing the sum and $d\varphi^2$ integration the result for Ω_φ reads

$$\Omega_\varphi = \frac{\varphi^2}{4g_1^2} - \frac{N_cN_f}{\pi^2}T\int dpp^2(\ln 2ch\theta^{(-)} + \ln 2ch\theta^{(+)}), \tag{12}$$

$$\theta^{(\mp)} = \frac{\sqrt{p^2+\varphi^2}\mp\mu}{2T}. \tag{13}$$

Next we expand Ω_φ in powers of φ in the vicinity of T_c. Expansion of the thermodynamic potential containing the second and the fourth order terms is called Ginzburg-Landau functional. The first order term is absent because of the mean-field stationarity condition $\partial\Omega_\varphi/\partial\varphi = 0$. The discussion of the interesting physics associated with the tricritical point and sixth order term is out of the scope of the present paper, see [1, 2, 19, 20].

With the expression (12) for the thermodynamic potential at hands the calculation of the derivatives $\partial^2\Omega_\varphi/\partial\varphi^2$ and $\partial^4\Omega_\varphi/\partial\varphi^4$ is rather straightforward. One has

$$2A(T) \equiv \left.\frac{\partial^2\Omega_\varphi}{\partial\varphi^2}\right|_{\varphi=0} = \frac{1}{2g_1^2} - \frac{N_cN_f}{2\pi^2}\int dpp(th\theta^{(-)} + th\theta^{(+)}). \tag{14}$$

Denoting $\theta^{(\mp)}$ at $T = T_c$ as $\theta_c^{(\mp)}$ one writes

$$th\theta^{(\mp)} \simeq th\theta_c^{(\mp)} + \left(\frac{T_c - T}{2T_c^2}\right)\frac{(p\mp\mu)}{ch^2\theta_c^{(\mp)}}. \tag{15}$$

Substitution of (15) into (14) leads after simple integration to the result

$$A(T) = \frac{1}{6}N_cN_fT_c^2\left(\frac{T-T_c}{T_c}\right) = T_c^2\left(\frac{T-T_c}{T_c}\right). \tag{16}$$

In arriving to (16) use was also made of the gap equation $\partial\Omega_\varphi/\partial\varphi = 0$ which yields cancellation of the $1/2g_1^2$ term. Strictly speaking the gap equation is $\partial\Omega/\partial\varphi = 0$, where Ω is the full thermodynamic potential (9) but as we shall see the corresponding corrections are small. Note that $A < 0$ at $T < T_C$ in line with the general theory of phase transitions.

Calculation of the fourth order derivative is almost as simple. As a preliminary remark we remind that for $N_f = 2$ chiral symmetry restoration at $\mu = 0$ proceeds via second order phase transition. On the other hand at $T = 0$ but with μ increasing the restoration of chiral symmetry occurs via first order phase transition [1, 2, 19, 20]. Correspondingly in the first case the coefficient of the quartic term has to be positive while in the second case -negative. The sign is changed at the tricritical point [1, 2, 19, 20] at which the sixth order term is important. Consideration of this point is beyond the scope of the present paper. Another remark is that in the NJL model the order of the phase transition depends upon the value of the cutoff [21]. We shall immediately see that indeed the sign of the fourth order term depends on the value of the chemical-potential μ and of the cutoff value Λ.

The nonvanishing at $\varphi = 0$ part of the fourth order derivative reads

$$4B(T,\mu) \equiv \frac{\partial^4\Omega_\varphi}{\partial\varphi^4} = \frac{3N_cN_f}{2\pi^2}\int dp\left\{\frac{th\theta^{(-)} + th\theta^{(+)}}{p} - \frac{1}{2T_c}\left(\frac{1}{ch^2\theta^{(-)}} + \frac{1}{ch^2\theta^{(+)}}\right)\right\}. \tag{17}$$

Simple manipulations yield

$$B(T,\mu)=\frac{9}{2\pi^2}\left\{\int_0^{\Lambda/T_c}dt\frac{tht}{t}\left(\frac{cht}{cht+ch\frac{\mu}{T_C}}\right)-1\right\}. \tag{18}$$

As expected the sign of B depends on the values of the chemical potential μ and cutoff Λ. Having in mind to demonstrate explicity the dependence on the cutoff we may resort to a rough estimate, namely

$$\int_0^{\Lambda/T_c}dt\frac{tht}{t}\left(\frac{cht}{cht+ch\frac{\mu}{T_C}}\right)<\int_0^{\Lambda/T_c}dt\frac{tht}{t}=th\frac{\Lambda}{T_c}\ln\frac{\Lambda}{T_c}-\int_0^{\infty}dt\frac{\ln t}{ch^2t}=$$

$$=th\frac{\Lambda}{T_c}\ln\frac{\Lambda}{T_c}+\ln\frac{4\gamma}{\pi}\simeq\ln\frac{4\gamma\Lambda}{\pi T_c}, \tag{19}$$

where $\gamma=e^C, C=0.5777...$ Typical cutoff value is $\Lambda\sim 800$ MeV [1, 2, 21].

Collecting pieces together we write down the Ginzburg-Landau functional for the chiral part of the thermodynamic potential ($N_c=3, N_f=2$);

$$\Omega_\varphi=T_c^2\left(\frac{T-T_c}{T_c}\right)\varphi^2+\frac{9}{2\pi^2}\left\{\int_0^{\Lambda/T_c}dt\frac{tht}{t}\left(\frac{cht}{cht+ch\frac{\mu}{T_C}}\right)-1\right\}\varphi^4. \tag{20}$$

We note that the above treatment was oversimplified in several points, in particular we assumed that the coupling g_1^2 was temperature independent, the quark current masses were set to zero, etc. These omissions important by themselves are hardly crucial for our main purpose which is the formulation and proof of the Anderson theorem.

4 Ginzburg-Landau potential for diquarks

Now we turn to the last term in Eq. (9). Expanding the logarithm in powers of Δ^2 we directly arrive at the desired Ginzburg-Landau functional. The term proportional to Δ^2 reads

$$\Omega^{(2)}_{\Delta\varphi}=\frac{\Delta^2}{4g_2^2}-\frac{1}{2}tr\frac{\Delta^2\Phi\Phi^+}{(\hat{p}_+-i\varphi\Lambda)(\hat{p}_-+i\varphi\Lambda)}. \tag{21}$$

Inverting the denominator and performing the trace over discrete indices one gets ($N_c=3$)

$$\Omega^{(2)}_{\Delta\varphi}=\frac{\Delta^2}{4g_2^2}-8\Delta^2T\sum_n\int\frac{d^3p}{(2\pi)^3}\frac{(p_4^2+\mathbf{p}^2+\mu^2+\varphi^2)}{R}, \tag{22}$$

where

$$R=p_4^4+2p_4^2(\mathbf{p}^2+\mu^2+\varphi^2)+(\mathbf{p}^2-\mu^2+\varphi^2)^2, \tag{23}$$

and the sum (22) is taken over fermionic Matsubara modes corresponding to p_4. Next step differs from the calculation of the φ^2 term presented in the previous section. Namely we assume that considering the Landau-Ginzburg region for the diquark condensate Δ one can neglect the contribution of antiparticles. This means that in the representation

$$\frac{p_4^2+\mathbf{p}^2+\mu^2+\varphi^2}{R}=\frac{1}{2}\left\{\frac{1}{p_4^2+(\sqrt{\mathbf{p}^2+\varphi^2}-\mu)^2}+\frac{1}{p_4^2+(\sqrt{\mathbf{p}^2+\varphi^2}+\mu)^2}\right\} \tag{24}$$

only the first term is kept. Then summation in (22) yields

$$\Omega^{(2)}_{\Delta\varphi} = \frac{\Delta^2}{4g_2^2} - \Delta^2 N(0) \int_{-\omega_D}^{\omega_D} d\xi \frac{1}{q} th\frac{q}{2T}, \tag{25}$$

where

$$\xi = p - \mu, \quad q = \sqrt{p^2 + \varphi^2} - \mu, \tag{26}$$

and ω_D is the Debye frequency while $N(0)$ is the density of states at the Fermi surface

$$N(0) = \frac{2}{\pi^2} \left(p^2 \frac{dp}{d\xi} \right)_F = \frac{2\mu^2}{\pi^2}. \tag{27}$$

The density of states (27) is four times larger than the BCS theory factor $\mu^2/2\pi^2$. This is due to color and flavor degrees of freedom, in 2SC there are red and green u and d quarks.

Next we expand the integrand in (25) in powers of $(T - T_c)$ and φ. This gives

$$\frac{1}{q} th\frac{q}{2T} \simeq \frac{1}{\xi} th\frac{\xi}{2T_c} + \left(\frac{T_c - T}{4T_c^2} \right) ch^{-2}\frac{\xi}{2T_c} + \frac{\varphi^2}{8\mu T_c^2} \left(\frac{1}{y} thy \right)', \tag{28}$$

with $y = \xi/2T$. The last term drops upon integration over the interval $[-\omega_D/2T_c, \omega_D/2T_c]$. Integration results also in cancellation of the first term with the term $\Delta^2/4g_2^2$ in (21) (because of the gap equation or equivalently from the definition of the critical temperature). Thus

$$\Omega^{(2)}_{\Delta} = \Delta^2 N(0) \left(\frac{T - T_c}{T_c} \right). \tag{29}$$

We see that up to φ^2 there is no dependence on the chiral condensate, therefore we dropped the index φ in (29). At this point one may notice the analogy with the Anderson theorem in BCS theory.

Next we consider the term proportional to Δ^4 in the expansion of the logarithm in (9). In this term the φ dependence will be neglected from the beginning. There is no need to present the details of the calculations which are simple and essentially repeat the derivation of the Δ^2 term with only two new technical points which are

$$\sum_n \frac{1}{(\omega_n^2 + \varepsilon^2)^2} = -\frac{1}{2\varepsilon} \frac{\partial}{\partial \varepsilon} \sum_n \frac{1}{\omega_n^2 + \varepsilon^2}, \quad \int_0^\infty \frac{dy}{y} \left(\frac{thy}{y} \right)' = -\frac{7\zeta(3)}{\pi^2}. \tag{30}$$

The result for $\Omega^{(4)}_{\Delta}$ reads

$$\Omega^{(4)}_{\Delta} = \Delta^4 N(0) \frac{7\zeta(3)}{16\pi^2 T_c^2}. \tag{31}$$

Thus the development of $\Omega_{\Delta\varphi}$ in powers of Δ and φ reads

$$\Omega_{\Delta} = -\Delta^2 N(0) \left\{ 1 - \frac{T}{T_c} - \frac{7\zeta(3)}{16\pi^2 T_c^2} \Delta^2 \right\}. \tag{32}$$

We note that with realistic values of Δ and μ [1] the numerical value of $\Omega_{\Delta\varphi}$ given by (32) is in perfect agreement with our estimate presented in [12].

Now we are tooled to formulate conclusions concerning the interplay of the two condensates. However we first turn from the Ginsburg-Landau region to the $T = 0$ case in order to demonstrate that the version of the Anderson theorem also holds at $T = 0$.

5 Renormalized Potential at $T = 0$

At $T = 0$ the formal development of the thermodynamic potential (9) in powers of Δ and φ is not appropriate. Instead we calculate the potential explicitly. The "φ-part" of it was evaluated in Sec. 3 exactly so we may just set $T = 0$ in (12) and get

$$\Omega_\varphi(T=0) = \frac{\varphi^2}{4g_1^2} - \frac{N_c N_f}{\pi^2}\int dp p^2\{\sqrt{p^2+\varphi^2} + (\mu - \sqrt{p^2+\varphi^2})\theta(\mu - \sqrt{p^2+\varphi^2})\}. \tag{33}$$

Next we turn to the last term in (9)

$$\Omega_{\Delta\varphi} = \frac{\Delta^2}{4g_2^2} - \frac{1}{2} tr \ln\left\{1 + \frac{\Delta\Delta^+\Phi\Phi^+}{(\hat{p}_+ - i\varphi\Lambda)(\hat{p}_- + i\varphi\Lambda)}\right\}. \tag{34}$$

Here use can be made of the standard trick; first one takes $\Delta\Omega_{\Delta\varphi}/\partial\Delta$, then performs integration over dp_4 which corresponds to $T = 0$, and then reconstructs $\Omega_{\Delta\varphi}(T = 0)$ by integration back over Δ. The result reads

$$\Omega_{\Delta\varphi}(T=0) = \frac{\Delta^2}{4g_2^2} - \frac{2}{\pi^2}\int dp p^2\{\sqrt{(\sqrt{p^2+\varphi^2}-\mu)^2+\Delta^2} + \sqrt{(\sqrt{p^2+\varphi^2}-\mu)^2+\Delta^2}\}. \tag{35}$$

As before the second term corresponding to antiparticles will be neglected. In order to avoid the onset of "irrelevant" terms of higher order in Δ we renormalize $\Omega_{\Delta\varphi}(T = 0)$ at certain scale $\kappa \gg \mu$ [22]. The renormalized coupling constant G_2^2 is defined as

$$\frac{1}{G_2^2} \equiv \left|\frac{\partial^2\Omega_{\Delta\varphi}(T=0)}{\partial\Delta^2}\right| \Delta = \kappa. \tag{36}$$

The renormalized potential reads

$$\Omega^r_{\Delta\varphi} = \frac{\Delta^2}{2G_2^2} - N(0)\int dp\{R_\Delta - \frac{\Delta^2}{2R_\kappa} + \frac{\Delta^2\kappa^2}{2R_\kappa^3}\}, \tag{37}$$

where

$$R_\alpha = \{(\sqrt{p^2+\varphi^2}-\mu)^2 + \alpha^2\}^{1/2}, \quad \alpha = \Delta, \kappa. \tag{38}$$

Performing integration and expansion in φ one gets

$$\Omega^r_{\Delta\varphi} = \frac{\Delta^2}{2G_2^2} - N(0)\Delta^2\left(\frac{3}{2} - \ln\frac{\Delta}{\kappa}\right) - N(0)\frac{\varphi^4}{4\mu^2}. \tag{39}$$

We see that correction of the order φ^2 is absent.

6 Assembling the Pieces

The main result of the paper is the Ginzburg-Landau functional with both condensates included. It reads

$$\Omega(\Delta,\varphi;\mu,T) = \varphi^2 T_c^2\left(\frac{T-T_c}{T_c}\right) + +\varphi^4\frac{9}{2\pi^2}\left\{\int_0^{\Lambda/T_c} dt\frac{tht}{t}\left(\frac{cht}{cht - ch\frac{\mu}{T_c}}\right) - 1\right\} +$$

$$\Delta^2 N(0)\left(\frac{T-T_c}{T_c}\right) + \Delta^4\frac{N(0)}{T_c^2}\frac{7\zeta(3)}{16\pi^2}. \tag{40}$$

The first two terms stem from the "chiral" part of the thermodynamic potential while the "mixed" part resulted in the last two terms with φ contribution suppressed (only terms of the order higher than φ^2 may appear). Comparing the coefficients of the first two terms with those of the second two one notices that the contribution of the chiral condensate is damped by a factor $T_c^2/\mu^2 \ll 1 (T_c \simeq 40$ MeV, $\mu \simeq 400$ MeV).

Therefore we arrive to the following two conclusions;

(i) The system of gap equations for chiral and diquark condensates actually decouples.

(ii) As soon as diquark condensate is formed the contribution of the chiral condensate to thermodynamic quantities becomes strongly suppresses.

In analogy with the BCS theory these statements form what may be somewhat loosely called the Anderson theorem.

Its validity is supported not only by numerical calculations within several modes [3]-[5] but also within the random matrix approach [7] based solely on the symmetry of the system.

Several assumptions and simplifications made on the way to the above conclusions were outlined in the text. They are the following. The current quark masses were neglected and the same critical temperature was taken for chiral and diquark condensates. The first assumption is a technical one and was made in order to avoid lengthy equations. It will be lifted in the next publication. The equality of the critical temperatures T_φ and T_Δ in the vicinity of the μ value where the diquark condensate arises was demonstrated by numerical calculations in [3]. This assumption is neither crucial and our final result (40) is easily generalized to the case of different T_φ and T_Δ. The expression (9) for the thermodynamic potential is quite general and is valid for a variety of the interaction models and different values of the coupling constants. The central step is the expansion of the logarithm in (9) up to Δ^2 and Δ^4 terms. This procedure is valid near the transition point but beyond the region of strong fluctuations.

The suppression of the chiral condensate by the factor T_c^2/μ^2 in the final expression (40) for Ω is due to the μ^2 factor in the density of quark states at finite chemical potential (see (27)).

Finally we want to stress the point which we consider most important for future amendment. This is the investigation of the role played by gluon fields [12]. Nonperturbative gluon fields introduce into the subject a new quantity - the correlation length T_g. It might be even speculated that the term Δ^3 in the Ginzburg-Landau functional will appear.

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